

Focus Theory of Choice – An axiomatized decision theory under uncertainty

**Peijun Guo
Faculty of Business Administration
Yokohama National University**

Agenda

- 1. Focus theory of choice (FTC)**
- 2. Axiomatization**
- 3. Resolving paradoxes**
- 4. Psychological evidences**
- 5. Concluding Remarks**

Focus theory of choice (FTC)

A decision maker (DM) is endowed with two distinct evaluation systems:

- a positive evaluation system (PES);
- a negative evaluation system (NES).

For the DM, one system is apparent and the other is latent. Which system is apparent is strongly related to the DM's personality traits (e.g., optimistic, pessimistic).

It can also be strongly influenced by the framing.

Cont'd

In the PES, for each lottery, the event which brings about a relatively high outcome with a relatively high probability has a relatively high accessibility. Such an event generates the individual's overall impression of this lottery. We call this **the positive focus of this lottery**.

Then, based on the positive foci of all lotteries, the best lottery is chosen.

Choosing an action owing the most attractive event.

Cont'd

In the NES, an event which leads to a relatively low outcome with a relatively high probability has a relatively high accessibility. This event will generate the impression of the lottery. We call this **the negative focus of this lottery.**

Then, based on the negative foci of all lotteries, the best lottery is chosen.

Choosing an action owing the most acceptable event.

Example: Decision under ignorance

The event which makes an action generate the highest outcome is the positive focus of this action because it is the most attractive event for this action;

The DM chooses such an action that produces the highest outcome from among all positive foci.

This is exactly the maximax criterion.

The event which makes an action yield the lowest outcome is the negative focus of this action because it is the most concerned event for this action; the DM then chooses from the negative foci the one with the highest outcome.

This is just the maximin criterion.

Axiomatization

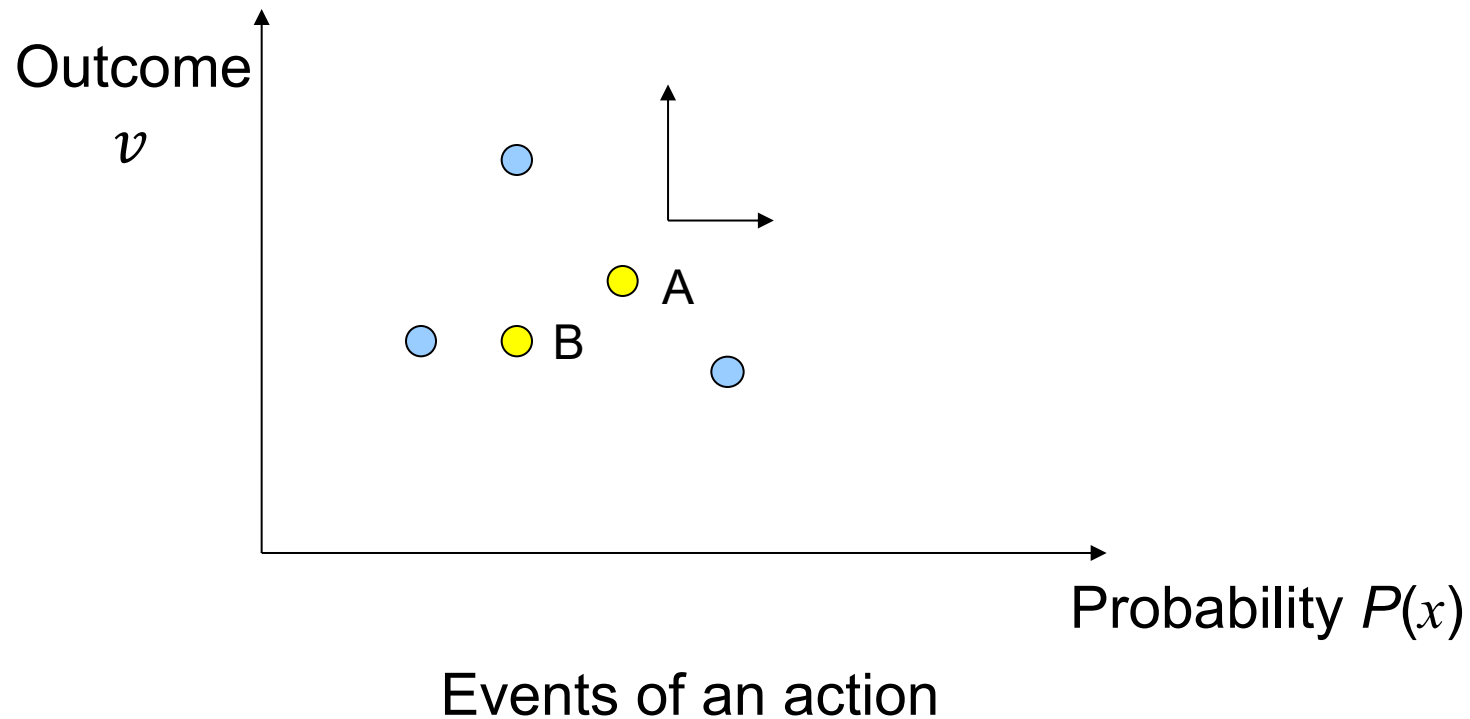
Positive Evaluation System (PES)

Axiom 1- Decidability: For each action, a DM can choose the most attractive event from its set of events.

Axiom 1 postulates that a DM is able to select the most attractive event from among all events of each action. Meanwhile, it implies that the most attractive event is not necessarily derived from a pairwise comparison where completeness and transitivity are needed.

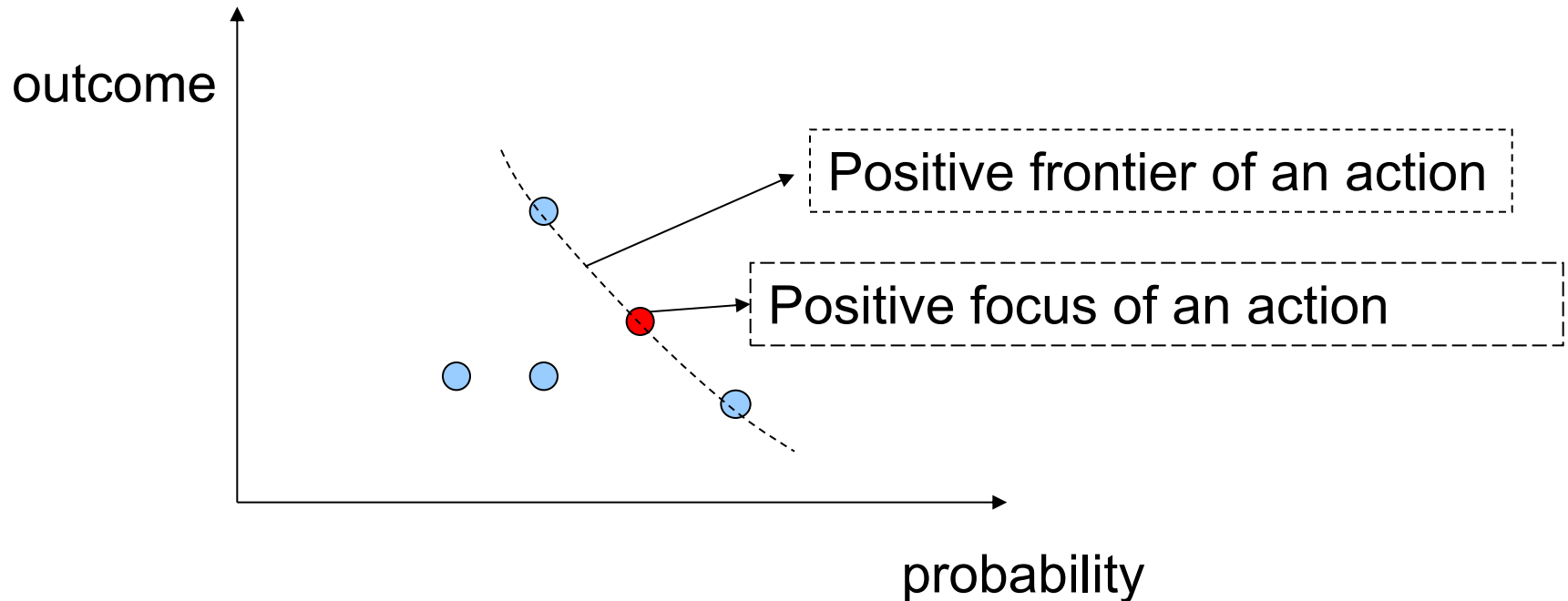
Cont'd

Axiom 2- Dominance:



Positive frontier of an action

Positive focus point of an action



Positive frontier of an action: the set of undominated events of an action.

Positive focus of an action: the most attractive event of an action.

The relative likelihood

If a function $\pi: S \rightarrow [0,1]$ satisfies $\max_{x \in S} \pi(x) = 1$,
then $\pi(x)$ is called a likelihood level function.

The relative likelihood function represents the relative position of likelihood.

$$\pi(x) = P(x)/\max P(x)$$

$P(x)$: the probability mass function

$$\pi(x) = f(x)/\max f(x)$$

$f(x)$: the probability density function

Satisfaction function

Satisfaction function: The normalized payoff function representing the relative position of payoff.

$$u(x, a_i) = v(x, a_i) / \max v(x, a_i)$$

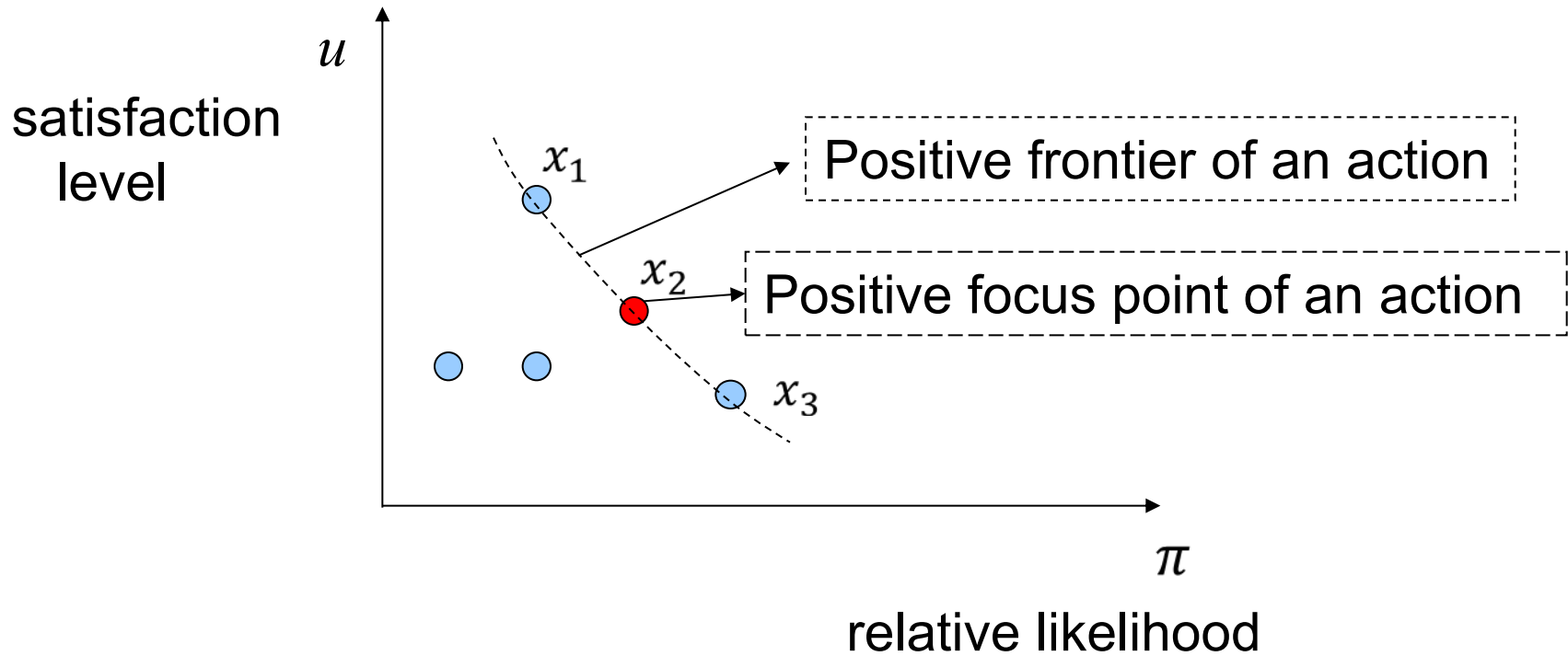
$u(x, a_i)$: satisfaction function

$v(x, a_i)$: payoff function

Both relative likelihood function and satisfaction function are exogenously given.

Positive frontier of an action

Positive focus point of an action



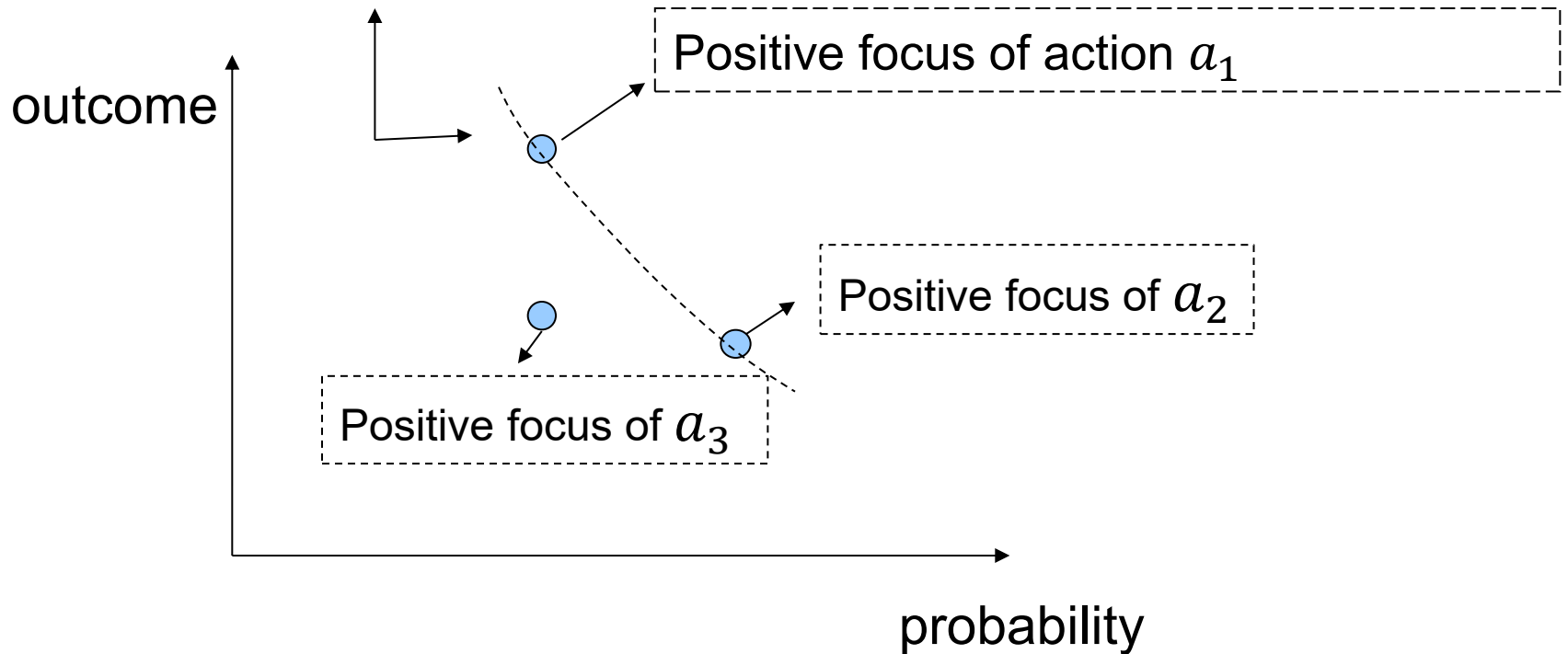
Representation theorem of the positive focus point (Theorem 1)

For x_2 , there exists a function $\pi(x) \wedge ((1/\varphi) * u(x, a_i))$

$$\begin{aligned} & \pi(x_2) \wedge ((1/\varphi) * u(x_2, a)) \\ &= \max \left(\begin{array}{l} \pi(x_1) \wedge ((1/\varphi) * u(x_1, a)), \pi(x_2) \wedge ((1/\varphi) * u(x_2, a)) \\ , \pi(x_3) \wedge ((1/\varphi) * u(x_3, a)) \end{array} \right) \end{aligned}$$

$\pi(x) \wedge (1/\varphi) * u(x, a)$: the attractiveness of the focus x of action a
 φ : the degree of emphasizing possible outcome for choosing the positive focus, endogenously derived from the choosing the positive focus of an action.

Dominance relationship between foci of two actions



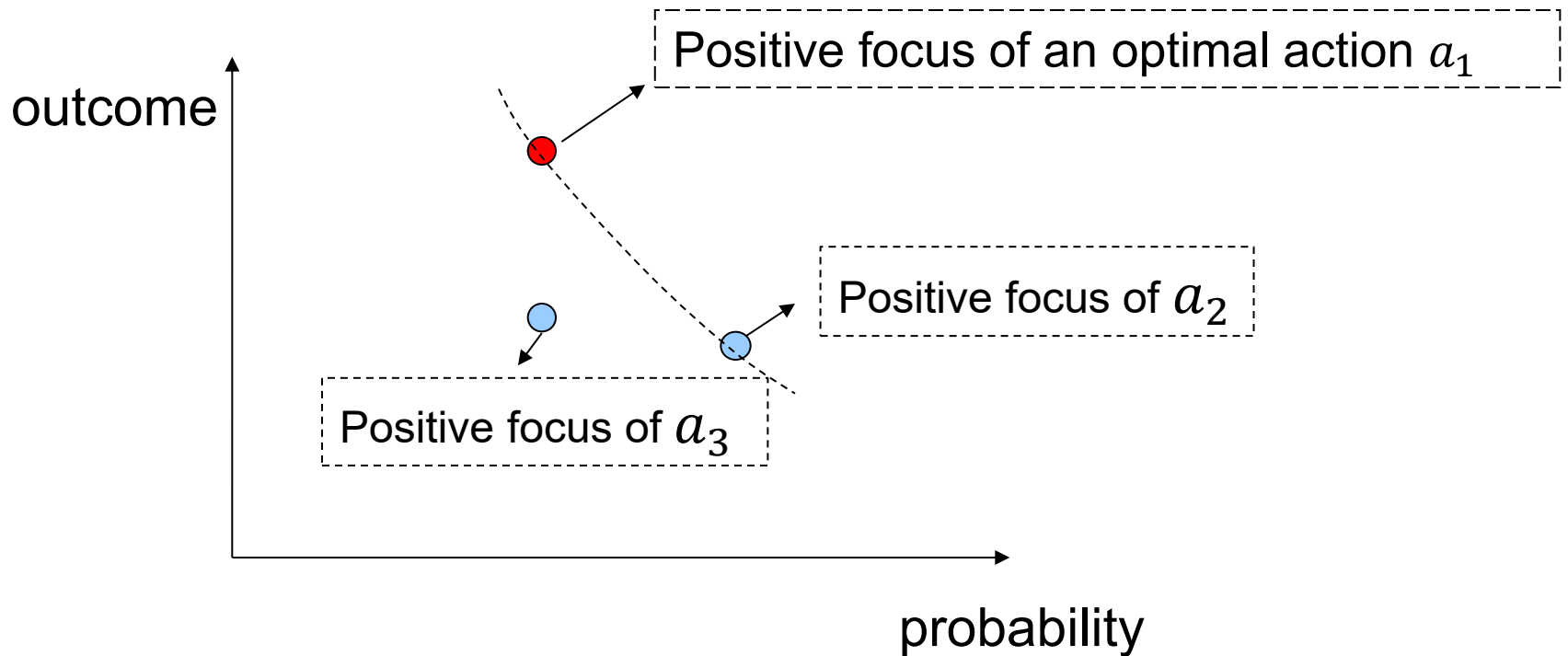
Axioms for the optimal action

Axiom 3- Decidability: A DM can choose the most preferred action.

It relaxes the assumptions of completeness and transitivity in standard economic theory and replace them by decidability. It means that a DM can determine his most-preferred action but there is no need to judge between any pair of actions. This assumption is intuitively appealing because in the real world the observable and observed action is usually the optimal action itself.

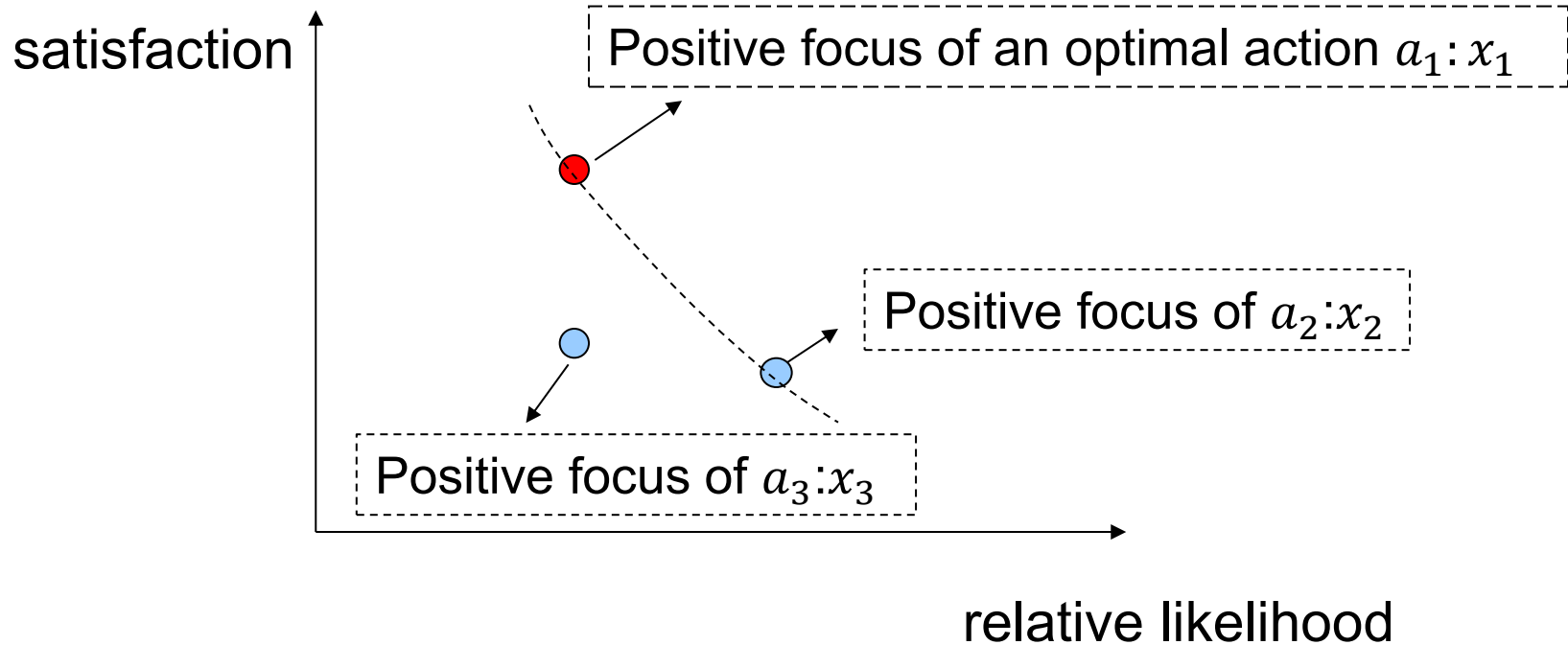
Axiom 4: Focus dominance

Axiom 4



Adjusted relative likelihood and satisfaction

Axiom 4

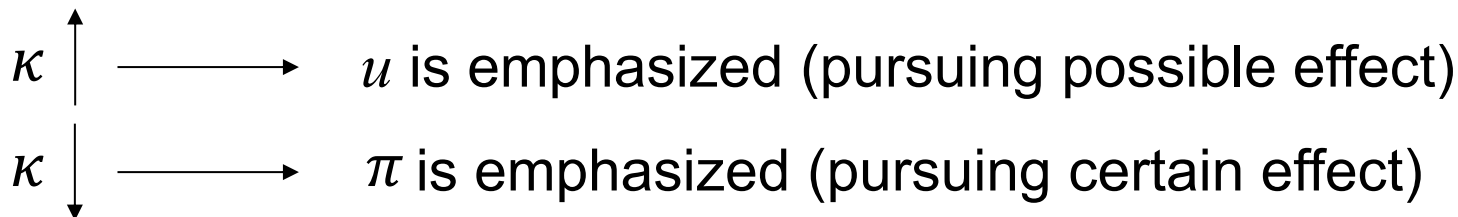


Representation theorem of an optimal action (Theorem 2)

If the optimal action a_1 satisfies axiom 4, then $\exists$ a function $\pi(x) \wedge (1/\kappa) * u(x, a)$ satisfying

$$\pi(x_1) \wedge ((1/\kappa) * u(x_1, a_1)) = \max(\pi(x_1) \wedge ((1/\kappa) * u(x_1, a_1)), \pi(x_2) \wedge ((1/\kappa) * u(x_2, a_2)))$$

$\min(\pi(x), (1/\kappa) * u(x, a))$: the attractiveness of an action a
 $\kappa > 0$: the degree of emphasizing the outcome, endogenously derived from choices of the optimal action.



Resolving anomalies

Ellsberg paradox

The St. Petersburg paradox

The Allais paradox

Preference Reversal

Event-Splitting Effect

Violations of Tail-Separability

Violations of Stochastic Dominance

Violations of Transitivity

Reflection effect

Common ratio effect

The evidence supporting FTC

Stewart, Hermens, & Matthews, Journal of Behavioral Decision Making 29 (2016) 116-136.

“We found very little systematic variation in eye movements over the time course of a choice or across the different choices.

The only exceptions were finding more (of the same) eye movements when **choice options were similar**, and an emerging gaze bias in which people looked more at the gamble they ultimately chose.

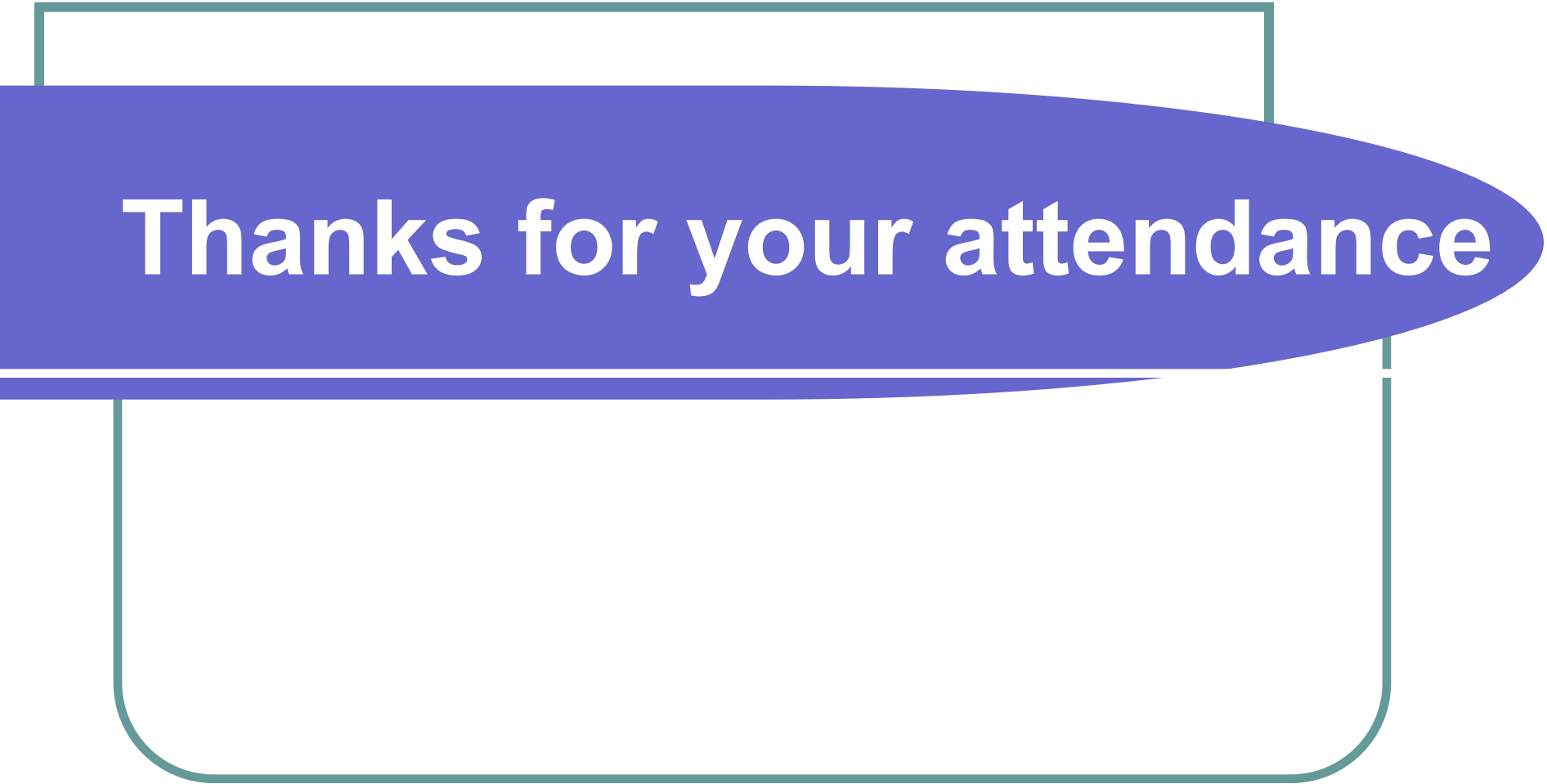
Cont'd

The eye movements made during a choice have a large relationship with the final choice, and this is mostly independent from the contribution of the actual attribute values in the choice options.

Eye movements tell us not just about the processing of attribute values but also are independently associated with choice. The pattern is simple—people choose the gamble they look at more often, independently of the actual numbers they see”.

Concluding remarks

1. We propose FTC which models and axiomatizes a decision-making procedure which are fundamentally different from the existing decision theories.
2. FTC provides a unified framework for handling decision making with risk or under ambiguity or under ignorance.
3. FTC can resolves many well-known anomalies, such as the St. Petersburg, Allais and Ellsberg paradoxes.
4. FTC provides a rigorous formal underpinning to understand the mechanism of human being decision making so that it can have comprehensive applications in industry, business, economy and social system.



Thanks for your attendance